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HYDRODYNAMIC SQUEEZE FILM LUBRICATION BETWEEN ROUGH PARALLEL PLATES: HIP-JOINT REPRESENTATION WITH MODIFIED BOUNDARY CONDITIONS

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ABSTRACT

Objectives: The objective of this paper is to analyze the unsteady hydrodynamic squeeze film lubrication between rough parallel plates with changed boundary conditions.

Methods/Statistical analysis: The stochastic model of Christensen and Tonder has been deployed here to evaluate the effect of surface roughness. Also the effect of roughness parameters on different moments is numerically modelled. The associated stochastically averaged Reynolds equation is solved to obtain the pressure distribution. The results obtained here are presented in graphical forms.

Findings: It has been conclusively establish that in general the transverse surface roughness effects the bearing performance adversely. Out of the three roughness parameters the standard deviation influences the bearing system significantly. Further a better situation is registered with reference to X_3 *axis.*

Application/Improvements: This theory can be developed to manufacture a suitable design for hip-joint.

Key words: Squeeze film, roughness, hydrodynamic lubrication, hip-joint, Reynolds equation, pressure.

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1. INTRODUCTION

Bio-tribology, the study of lubrication, wear and friction within the body has become a topic of encountered debilitating disease and trauma that destroy function of the joints. The hip-joint is a complex joint system which carries much of the weight of the body. It has sophisticated interaction between cartilage, bone, synovial fluid and other connected tissues. Due to degeneration of the joints, the use of artificial hip-joints has become necessary. With an increasing number of the ageing population requiring total hip replacements at a younger age, it is necessary to try and improve the design and longevity of the implant. A highly successful surgical procedure has been to replace the joint with an artificial equivalence which alleviates dysfunction and pain.

The tribological performance of an artificial hip-joint has become a strong topic of discussion. The principle causes for failure are adverse short and long term reactions to wear debris and high frictional torque in the case of poor lubrication that may cause loosening of the implant. Thus, using experimental and theoretical approaches models have been developed to evaluate lubricate on performance under standardized conditions.

In[1] studied lubrication mechanism of hip joint replacement prostheses. A laboratory study has been made of the lubrication and friction characteristics of a plastic on metaland a metal on metal total hip joint replacement prostheses. The results established the effect of lubricant, speed, and load on the performance of the joints. Under a 560 lb load the coefficient of friction of the plastic to metal joint was lower than that of the metal to metal at speeds up to the equivalent of fast walking. Bovine serum and synovial fluid, as well as human serum albumin were found to be good lubricants of both types of prostheses. The frictional force produced by the metal on metal prosthesis increased linearly with load, in both the dry and lubricated states. The results proceed that at low physiological loads the effort required to articulate the prostheses were comparable while at higher loads the friction force of the metal to plastic was significantly lower.

In[2] analyzed the effects of proteins on the friction and lubrication of artificial joints. The tribological testing of artificial hip and knee joints in the laboratory has been ongoing for several decades. This work has been carried out in an attempt to simulate the loading and motion conditions applied in vivo and, therefore, the potential for the success of the joint. However, several different lubricants have been used in these tests. The work documented in Scholes et:al paper[2] compared results obtained using different lubricants and made suggestions for further works in this direction.

In[3]discussed graphitic tribological layers in metal-on-metal hip replacements. Arthritis is a leading cause of disability, and when nonoperative methods have failed, a prosthetic implant is a cost-effective and clinically successful treatment. Metal-on-metal replacements are an attractive implant technology, a lower-wear alternative to metal-on-polyethylene devices. This paper dealt with the evidence for graphitic material in the tribological layer in metal-on-metal hip replacements retrieved from patients. As graphite is a solid lubricant, its presence helped to explain why these components exhibited low wear and suggested methods of improving their performance; simultaneously, this raised the issue of the physiological effects of graphitic wear debris.

In[4] deals with the observation of model synovial fluid lubricating mechanisms. This paper examined the fundamental mechanisms of synovial fluid lubrication in artificial joints. The results indicated that two types of film were formed; a boundary layer of adsorbed protein molecules, which were augmented by a high-viscosity fluid film generated by hydrodynamic effects. The high-viscosity film was due to inlet aggregation of protein molecules forming a

gel which was entrained into the contact preferentially at low speeds. As the speed increased this gel appeared to shear thin, giving much lower lubricant film thickness.

In[5] discussed the matter of synovial fluid lubrication: implications for metal-on-metal hip tribology. This which they reviewed the accepted lubrication models for artificial hips and presented a new concept to explain film formation with synovial fluid. The implications of this new mechanism for the tribological performance of new implant designed and the effect of patient synovial fluid properties were discussed.

In[6] studied synovial fluid lubrication: the effect of protein interactions on adsorbed and lubricating Films. In this work examined how the protein content of six model synovial fluids affected film formation under static and rolling conditions and if the changes in properties could be correlated.

In[7] viscous oscillatory fully developed flow of stratified fluid in a long vertical narrow rectangular channel with one side being pervious and the other side being impervious is considered. The outcome of this investigation reveals that the stratification effects reverse the flow near the center of the channel while magneto hydrodynamic effects make the flow linear closer to the plates.

In[8] non dimensional equation of resistance coefficient with Reynolds number of porous medium flow has been studied recently. Experimental results were compared with Darcy's equation and the validity of this equation was established. The derived equations were suggested to be applied in porous medium flow such that the velocity of flow could be determined from which discharge through porous medium can be estimated.

In[9] investigated the squeezing hydrodynamics of a lubricant between two rough parallel plates: hip-joint representation.The stochastic model of Christensen and Tonder was deployed here to evaluate the effect of surface roughness. Also the effect of roughness parameters on different moments was numerically modelled. The results established that the standard deviation associated with roughness had significant impact. Further it was observed that the situation remained relatively better in the case of negatively skewed roughness. This effect advanced when variance (-ve) occurs.

Here it has been proposed to study the hydrodynamics squeeze film lubrication between rough parallel plates in the context of hip joint. A new set of boundary conditions has been introduced.

2. MATERIALS AND METHODS

With usual assumptions of fluid film lubrication the velocity profiles are governed by the equations as presented in[10]. Besides, following boundary conditions are adopted in[10] there is

$$\begin{aligned}
\mathbf{u} &= \left(-\frac{\sigma^2}{4\mu}, -\frac{\sigma^2}{4\mu}, -\frac{\sigma^2}{4\mu}\right) \text{ on } x_3 = 0 \\
\mathbf{u} &= \left(U_{1,} -\frac{\sigma^2}{4\mu}, U_{2,} -\frac{\sigma^2}{4\mu}, U_3 -\frac{\sigma^2}{4\mu}\right) on\ x_3 = h
\end{aligned} \tag{1}$$

where h is the film thickness, the distance between surfaces. In the present situation it has been sought to investigate the equations

$$\frac{\partial p}{\partial x_1} = \mu \frac{\partial^2 u_1}{\partial x_3^2}$$

and

$$\frac{\partial p}{\partial x_2} = \mu \frac{\partial^2 u_2}{\partial x_3^2}$$

the resulting velocity profiles are obtained after integrating with respect to x_3,

$$u_1 = \frac{\partial p}{\partial x_1}\left(\frac{x_3^2 - x_3 h}{2\mu}\right) + \frac{U_1 x_3}{h} - \frac{\sigma^2}{4\mu}$$

$$u_2 = \frac{\partial p}{\partial x_1}\left(\frac{x_3^2 - x_3 h}{2\mu}\right) + \frac{U_2 x_3}{h} - \frac{\sigma^2}{4\mu} \tag{2}$$

Conservation of mass is now taken into account by integrating equation

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0$$

across the layer, resulting in

$$\int_0^{h(x_1,x_2,t)} \frac{\partial u_3}{\partial x_3} \partial x_3 = -\int_0^{h(x_1,x_2,t)} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}\right) \partial x_3 \tag{3}$$

The film thickness is dependent on x_1, x_2 and t.Applying Liebnitz's theorem of integration and also using boundary conditions one arrives at

$$\frac{\partial}{\partial x_1} \int_0^{h(x_1,x_2,t)} u_1(x_1,x_2,x_3,t)\partial x_3 = \int_0^h \frac{\partial u_1}{\partial x_1}\partial z + u_1(x_1,x_2,h,t)\frac{\partial h}{\partial x_1} \tag{4}$$

$$\frac{\partial}{\partial x_2} \int_0^{h(x_1,x_2,t)} u_2(x_1,x_2,x_3,t)\partial x_3 = \int_0^h \frac{\partial u_2}{\partial x_2}\partial z + u_2(x_1,x_2,h,t)\frac{\partial h}{\partial x_2} \tag{5}$$

But the integrals on LHS of above equations are,

$$\int_0^{h(x_1,x_2,t)} u_1(x_1,x_2,x_3,t)\partial x_3 = -\frac{\partial p}{\partial x_1}\frac{h^3}{12\mu} + \frac{U_1 h}{2} - \frac{\sigma^2 h}{4\mu}$$

$$\int_0^{h(x_1,x_2,t)} u_2(x_1,x_2,x_3,t)\partial x_3 = -\frac{\partial p}{\partial x_2}\frac{h^3}{12\mu} + \frac{U_2 h}{2} - \frac{\sigma^2 h}{4\mu} \tag{6}$$

In view of the stochastic model of Chirstonsen and Tonder[11,12,13] these equations transform to

$$\int_0^{h(x_1,x_2,t)} u_1(x_1,x_2,x_3,t)\partial x_3 = -\frac{\partial p}{\partial x_1}\frac{G(h)}{12\mu} + \frac{U_1 h}{2} - \frac{\sigma^2 h}{4\mu}$$

$$\int_0^{h(x_1,x_2,t)} u_2(x_1,x_2,x_3,t)\partial x_3 = -\frac{\partial p}{\partial x_2}\frac{G(h)}{12\mu} + \frac{U_2 h}{2} - \frac{\sigma^2 h}{4\mu}$$

where $G(h) = h^3 + 3\sigma^2 h + 3\alpha^2 h + 3\alpha h^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon$

σ : standard deviation, α : variance, ε :skewness, h : film thickness

Putting these values in equation (3), with the help of equation (4), (5) and resorting to modified boundary conditions,

$$u_1(x_1, x_2, x_3, t) = U_1 - \frac{\sigma^2}{4\mu}$$

and

$$u_2(x_1, x_2, x_3, t) = U_2 - \frac{\sigma^2}{4\mu}$$

one is led to

$$U_3 - \frac{\sigma^2}{4\mu} - \frac{U_1}{2}\frac{\partial h}{\partial x_1} - \frac{U_2}{2}\frac{\partial h}{\partial x_2} = \frac{\partial}{\partial x_1}\left\{\frac{G(h)}{12\mu}\frac{\partial p}{\partial x_1}\right\} + \frac{\partial}{\partial x_2}\left\{\frac{G(h)}{12\mu}\frac{\partial p}{\partial x_2}\right\} \tag{7}$$

Rewriting this relation using Kinematic boundary conditions on the surface $x_3 = h$ one gets

$$U_3 = \frac{\partial h}{\partial t} + U_1\frac{\partial h}{\partial x_1} + U_2\frac{\partial h}{\partial x_2} + \frac{\sigma^2}{2\mu} \tag{8}$$

Making use of (8) in (7), one arrives at,

$$\frac{\partial}{\partial x_1}\left\{\frac{G(h)}{12\mu}\frac{\partial p}{\partial x_1}\right\} + \frac{\partial}{\partial x_2}\left\{\frac{G(h)}{12\mu}\frac{\partial p}{\partial x_2}\right\} = \frac{\partial h}{\partial t} + \frac{U_1}{2}\frac{\partial h}{\partial x_1} + \frac{U_2}{2}\frac{\partial h}{\partial x_2} + \frac{\sigma^2}{4\mu}$$

In general form, this gets converted to

$$\nabla \bullet \left(\frac{G(h)}{12\mu}\nabla P\right) = \nabla \bullet \left(\frac{h\vec{U}}{2}\right) + \frac{\sigma^2}{4\mu} + \frac{\partial h}{\partial t} \tag{9}$$

where $\vec{U}$ is the surface velocity vector.

2.1 Reynolds Equation Applied to a Hip Joint

For modeling the Hip-joint the following assumptions are made,

- the fluid is isoviscous (Newtonian)
- the cup is positioned horizontally
- walking cycle imposed is based on the Bergmann walking cycle

Synovial fluid and blood plasma are known to have non-Newtonian fluid properties. But studies have concluded that at high shear rates, an isoviscous assumption is valid. The acetabular cup is anatomically positioned at 45^0; however, the contact mechanics model can be developed in a horizontal position. The Bergmann walking cycle consists of a loading pattern which is double-peaked. This has resulted from studies conducted by Bergmann et al. (1995)[14] on the influence of heel strike on the loading of the hip joint. The coordinate system that best describes the hip joint is the spherical coordinate system. Accordingly, expanding equation (9) using spherical coordinates, one comes to the conclusion that, for equation (9),

$$\nabla P = \frac{\partial P}{\partial R}\hat{R} + \frac{1}{R}\frac{\partial P}{\partial \theta}\hat{\theta} + \frac{1}{R\sin\theta}\frac{\partial P}{\partial \phi}\hat{\phi}$$

since

$$\frac{\partial P}{\partial R} = 0,$$

Making use of divergence formula for spherical coordinates one concludes that

$$\nabla \bullet \left(\frac{G(h)}{12\mu}\nabla P\right) = \frac{1}{R^2}\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{G(h)}{12\mu}\sin\theta\frac{\partial P}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial}{\partial\phi}\left(\frac{G(h)}{12\mu}\frac{\partial P}{\partial\phi}\right)\right\} \tag{10}$$

For RHS of equation (9), one derives that

$$\nabla \bullet \left(\frac{h\vec{U}}{2}\right) = \frac{1}{2R\sin\theta}\left[h\cos\theta\, U_\theta + h\sin\theta\frac{\partial U_\theta}{\partial\theta} + U_\theta\sin\theta\frac{\partial h}{\partial\theta} + h\frac{\partial U_\phi}{\partial\phi} + U_\phi\frac{\partial h}{\partial\phi}\right] \tag{11}$$

Substituting (10) and (11) in (9), one arrives at

$$\frac{1}{R^2}\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{G(h)}{12\mu}\sin\theta\frac{\partial P}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial}{\partial\phi}\left(\frac{G(h)}{12\mu}\frac{\partial P}{\partial\phi}\right)\right\}$$
$$= \frac{1}{2R\sin\theta}\left[h\cos\theta\, U_\theta + h\sin\theta\frac{\partial U_\theta}{\partial\theta} + U_\theta\sin\theta\frac{\partial h}{\partial\theta}\right.$$
$$\left. + h\frac{\partial U_\phi}{\partial\phi} + U_\phi\frac{\partial h}{\partial\phi}\right] + \frac{\partial h}{\partial t} + \frac{\sigma^2}{4\mu} \tag{12}$$

where the surface velocity components, U_θ and U_ϕ are given by

$$U_\theta = -R\omega_{x_1}\sin\phi + R\omega_{x_2}\cos\phi$$

$$U_\phi = -R\omega_{x_1}\cos\phi\cos\theta - R\omega_{x_2}\sin\phi\cos\theta + R\omega_{x_1}\sin\phi + R\omega_{x_3}\sin\theta$$

Substituting these relations in equation (12) and simplifying, one comes across

$$\sin\theta\frac{\partial}{\partial\theta}\left(G(h)\sin\theta\frac{\partial p}{\partial\theta}\right) + \frac{\partial}{\partial\phi}\left(G(h)\frac{\partial p}{\partial\phi}\right)$$
$$= 6\mu R^2\sin\theta\left(-\omega_{x_1}\sin\phi\sin\theta\frac{\partial h}{\partial\theta} + \omega_{x_2}\cos\phi\sin\theta\frac{\partial h}{\partial\theta} - \omega_{x_1}\cos\phi\cos\theta\frac{\partial h}{\partial\phi}\right.$$
$$\left. - \omega_{x_2}\sin\phi\cos\theta\frac{\partial h}{\partial\phi} + \omega_{x_3}\sin\theta\frac{\partial h}{\partial\phi} + 2\sin\theta\frac{\partial h}{\partial t} + 2\sin\theta\frac{\sigma^2}{4\mu}\right) \tag{13}$$

If the axes are shifted and the $x_1 - x_3$ plane lies in the horizontal position with x_2 in the vertical direction, the last equation can be simplified with rotation about the x_3 axis to obtain

$$\sin\theta \frac{\partial}{\partial\theta}\left(G(h)\sin\theta \frac{\partial p}{\partial\theta}\right) + \frac{\partial}{\partial\phi}\left(G(h)\frac{\partial p}{\partial\phi}\right) = 6\mu R^2 \sin^2\theta \left(\omega_{x_3}\frac{\partial h}{\partial\phi} + 2\frac{\partial h}{\partial t} + 2\frac{\sigma^2}{4\mu}\right)$$

Treating θ and t as constants it is found that,

$$\frac{\partial}{\partial\phi}\left(\left(h^3 + 3\sigma^2 h + 3\alpha^2 h + 3\alpha h^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon\right)\frac{\partial p}{\partial\phi}\right) = 6\mu R^2 \sin^2\theta \left(\omega_{x_3}\frac{\partial h}{\partial\phi} + \frac{\sigma^2}{2\mu}\right)$$

The dimensionless form of the above equation turns out to equation

$$\frac{\partial}{\partial\overline{\phi}}\left(\left(1 + 3\overline{\sigma}^2 + 3\overline{\alpha}^2 + 3\overline{\alpha} + 3\overline{\sigma}^2\overline{\alpha} + \overline{\alpha}^3 + \overline{\varepsilon}\right)\frac{\partial\overline{p}}{\partial\overline{\phi}}\right) = 6\sin^2\theta\,\omega_{x_3} + 3\sin^2\theta\,\overline{\sigma}^2 \tag{14}$$

where,

$\overline{\sigma}$ is the standard deviation in dimensionless form,

$\overline{\alpha}$ is non-dimensional variance,

$\overline{\varepsilon}$ is the skewness in dimensionless form,

$\overline{p}$ is non-dimensional pressure,

$\overline{\phi}$ is non-dimensional rotation.

Theassociated dimensionless boundary conditions are,

$$\overline{P} = 0 \quad at\ \overline{\phi} = 2\pi$$

$$\overline{P} = \omega_{x_3}\sin\theta \quad at\ \overline{\phi} = \pi \tag{15}$$

Integration of (13) in view of (14) leads to the expression for non- dimensional pressure distribution as

$$\overline{P} = \frac{3\sin^2\theta}{\left(1 + 3\overline{\sigma}^2 + 3\overline{\alpha}^2 + 3\overline{\alpha} + 3\overline{\sigma}^2\overline{\alpha} + \overline{\alpha}^3 + \overline{\varepsilon}\right)}\left(\overline{\phi}^2 - 4\pi\overline{\phi} + 3\pi^2\right)\left(\omega_{x_3} + \frac{\overline{\sigma}^2}{2}\right) + \omega_{x_3}\sin\theta \tag{16}$$

3. RESULT AND DISCUSSION

It is easily noticed that the pressure distribution is governed by the equation (16). It depends upon various parameters such as $\overline{\sigma}$, $\overline{\alpha}$, $\overline{\varepsilon}$, $\overline{\phi}$, ω_{x_3} and θ. This expression makes it clear that a judicious choice of velocity in the x_3 direction may lead to an overall improve situation. A comparision of equation (16) with that of equation (15) in [9] suggest that nominally increased pressure is registered here due to the modification of the boundary conditions. In the light of

the involvement of the factor $\left(\omega_{x_3} + \bar{\sigma}^2\right)$ the graphs, which are shown in figure 1-8, are generated for the pressure distribution.

It is seen that the standard deviation introduces an adverse effect by lowering pressure however negatively skewed roughness results in increased pressure. Same is the trend of pressure with respect to variance (-ve). The graphical representations indicate that the effect of rotation along x_3 direction remains predominant for this type of bearing system.

4. CONCLUSION

It is clearly observed that the modification of boundary conditions turns in a nominally increased pressure distribution. This study reveals that for any type of improvement, the standard deviation is required to be kept in minimum even if the velocity in x_3 direction is suitably chosen. Therefore, while designing the ball-socket bearing system, the roughness aspects must be given due consideration because even the negatively skewed roughness can increase the pressure, which may be further augmented due the occurrence of variance (-ve). Certainly, this study may be of some help particularly when the system has run for a long time.

It was deemed appropriate to modify the boundary conditions incorporating the roughness parameter for deriving a better hip-joint system. Therefore, the positive effect of negatively skewed roughness and variance (-ve) can be channelized while designing the hip- joint bearing system.

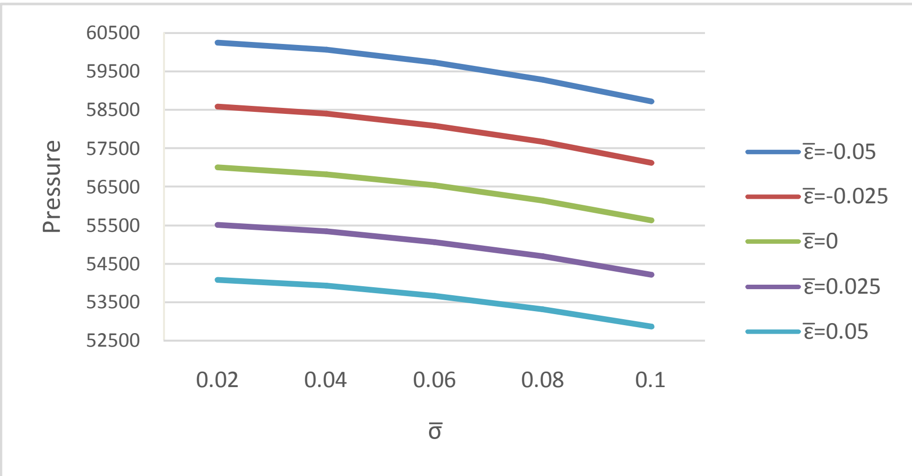

Figure 1 Variation of Pressure distribution with respect to $\bar{\sigma}$ and $\bar{\varepsilon}$

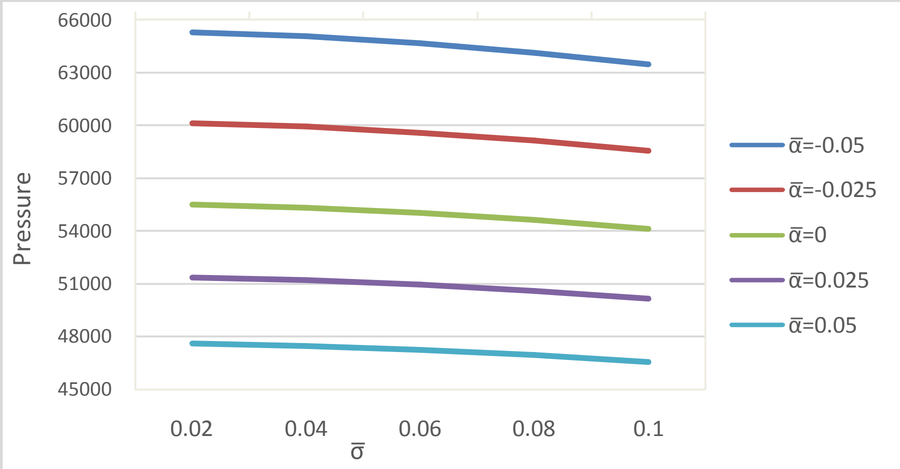

Figure 2 Variation of Pressure distribution with respect to $\overline{\sigma}$ and $\overline{\alpha}$

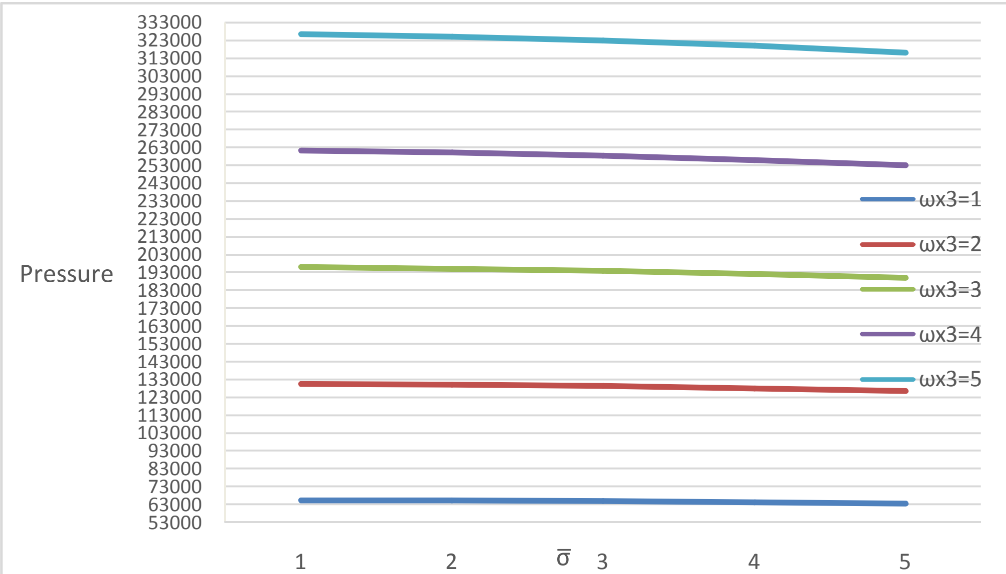

Figure 3 Variation of Pressure distribution with respect to $\overline{\sigma}$ and ω_{x_3}

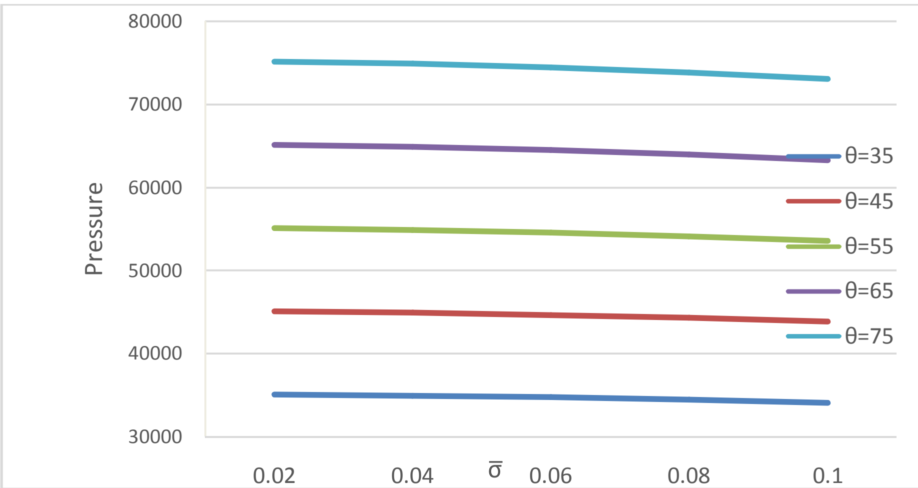

Figure 4 Variation of Pressure distribution with respect to $\overline{\sigma}$ and θ

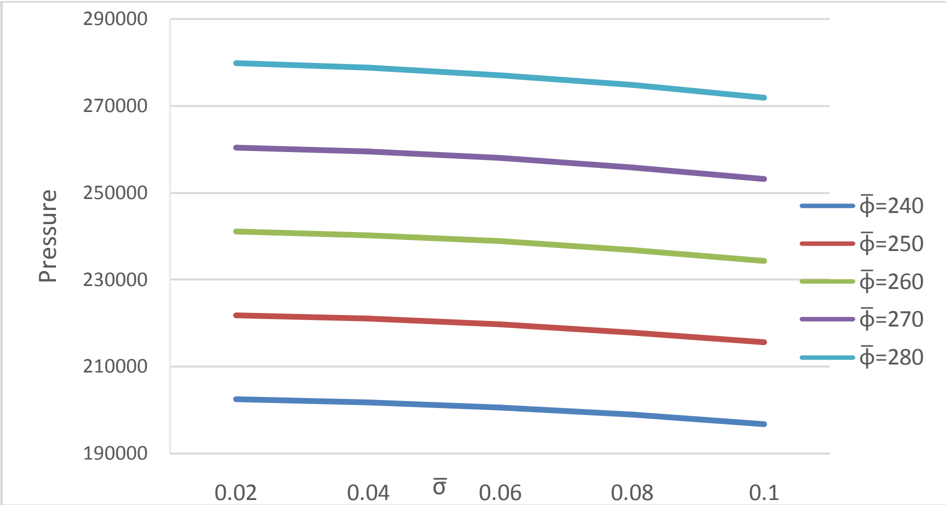

Figure 5 Variation of Pressure distribution with respect to $\bar{\sigma}$ and $\bar{\phi}$

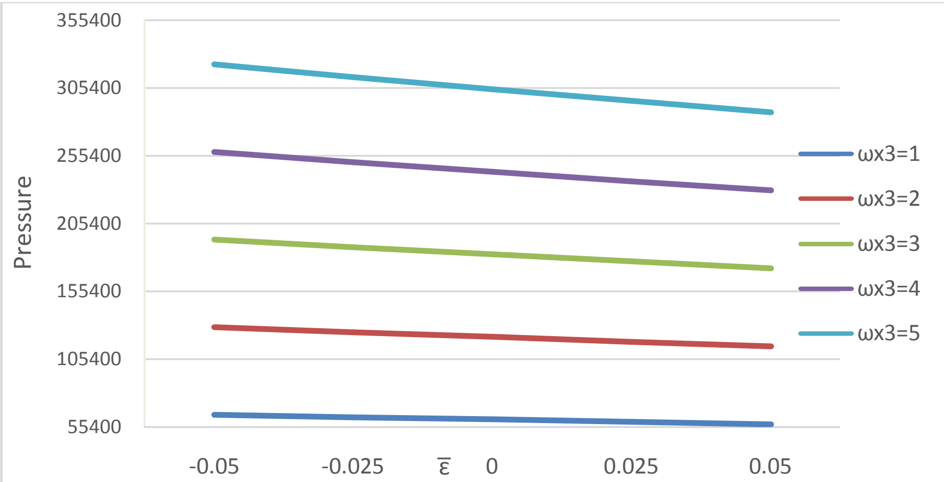

Figure 6 Variation of Pressure distribution with respect to $\bar{\varepsilon}$ and ω_{x_3}

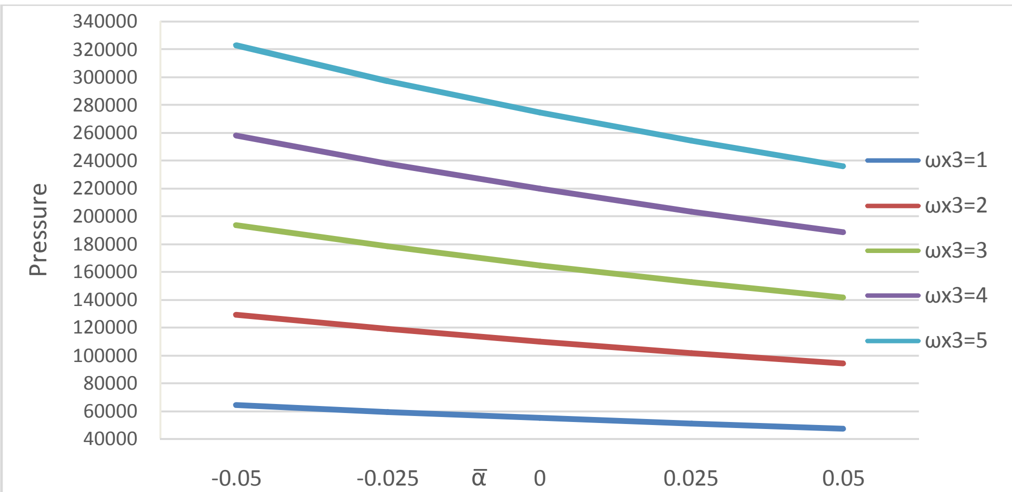

Figure 7 Variation of Pressure distribution with respect to $\bar{\alpha}$ and ω_{x_3}

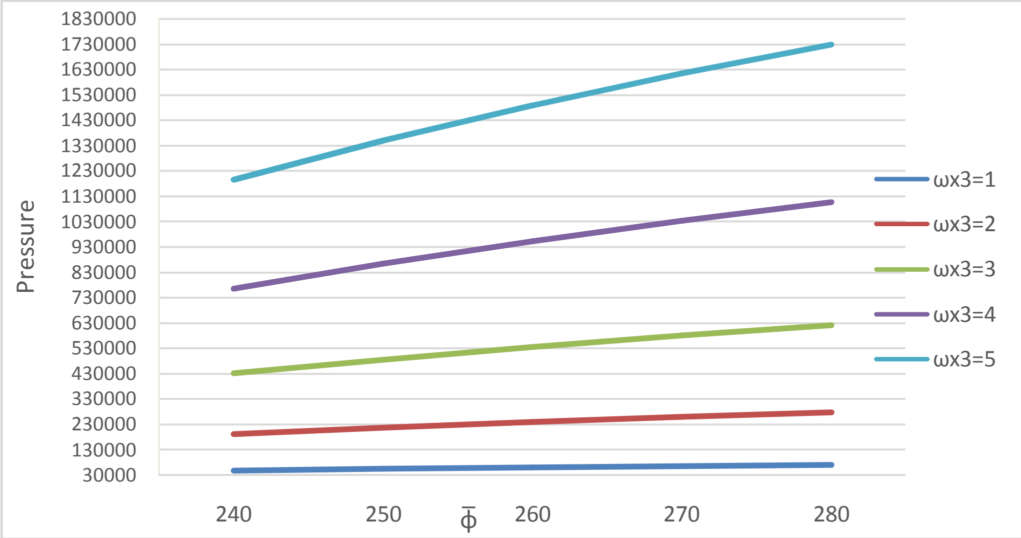

Figure 8 Variation of Pressure distribution with respect to ω_{x_3} and $\bar{\phi}$

5. ACKNOWLEDGEMENT

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REFERENCES

[1] B. Weightman, S. Simon, I. Paul, R. Rose and E. Radin ‘Lubrication Mechanism of Hip Joint Replacement Prostheses’. J. of Lubrication Tech volume 94 Issue 2(1972), 131-135.

[2] Scholes SC, Unsworth A. ‘The effects of proteins on the friction and lubrication of artificial joints’. ProcInstMechEng H. 2006 Aug;220(6):687-93.

[3] Y. Liao, R. Pourzal,M. A. Wimmer,J. J. Jacobs1,A. Fischer, L. D. Marks ‘Graphitic Tribological Layers in Metal-on-Metal Hip Replacements’. Science 23 Dec 2011:Vol. 334, Issue 6063, pp. 1687-1690.

[4] Connor Myant, Philippa Cann ‘In contact observation of model synovial fluid lubricating mechanisms’ Tribology International,Volume 63, July 2013, Pages 97–104.

[5] Connor Myant, Philippa Cann ‘On the matter of synovial fluid lubrication: Implications for Metal-on-Metal hip tribology’. Journal of the Mechanical Behavior of Biomedical Materials,Volume 34, June 2014, Pages 338–348.

[6] Maria Parkes,ConnorMyant,Philippa M. Cann,Janet S.S. Wong ‘Synovial Fluid Lubrication: The Effect of Protein Interactions on Adsorbed and Lubricating Films’. Biotribology Volumes 1–2, March–June 2015, Pages 51–60.

[7] L. PrasannaVenkatesh, S. Ganesh, K. B. Naidu,“Magnetohydrodynamic Viscous Oscillatory Stratified Flow in a Vertical Narrow Channel with a Porous Wall”,Indian Journal of Science and Technology,2015 Feb, 8(3), Doi no: 10.17485/ijst/2015/v8i3/59975.

[8] M. Thiruvengadam, G. N. Pradeep Kumar, D. Sivakumar, C. Mohan,“Non-Dimensional Equation of Resistance Coefficient with Reynolds Number of Porous Medium Flow”,Indian Journal of Science and Technology,2016 Mar, 9(11), Doi no:10.17485/ijst/2016/v9i11/89401.

[9] 9. Hemangini R. Jani, H. C. Patel,G. M. Deheri ‘A study on the squeezing hydrodynamics of a lubricant between two rough parallel plates: Hip-Joint representation’. International Journal of Innovation and Scientific Research, Volume 24, Issue 2,June 2016, Pages 359-372.

[10] ShatishRamjee ‘Numerical Analysis of Lubrication in an Artificial Hip Joint’ PhD thesis, Department of Chemical Engineering, University of Pretoria, Pretoria, 2008.

[11] Christensen H. and K. Tonder, 'The hydrodynamic lubrication of rough bearing surfaces of finite width'. Journal of Tribology 93.3 (1971): 324-329.
[12] Christensen H. and K. Tonder, 'The hydrodynamic lubrication of rough journal bearings'. Journal of Tribology 95.2 (1973): 166-172.
[13] Tonder K. and H. Christensen 'Waviness and roughness in hydrodynamic lubrication'. Proceeding of the Institution Mechanical Engineers 186.1 (1972): 807-812.
[14] Bergmann, G.; Kneggendorf, H.; Graichen, F. and Rohlmann, A. 'Influence of shoe and heel strike on the loading of a hip joint',1995 Journal of biomechanics, Volume 28 Issue 7.
[15] Sundarammal Kesavan & Santhana Krishnan N, Surface Roughness Effects on Squeeze Film Behavior in Porous Transversely Triangular Plates with Couple Stress Fluid. *International Journal of Mechanical Engineering and Technology*, 3(2), 2012, pp. 1–12
[16] Monayya Mareppa and S Santosh. Micropolar Fluid Lubrication of Finite Partial Rough Porous Journal Bearings with Squeeze Effect, *International Journal of Mechanical Engineering and Technology*, 6(11), 2015, pp. 258-279.